



Subject Name: LAPLACE TRANS FORMS ,NUMERICAL METHODS&COMPLX VARIABLES

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Year and Sem, Department: II & II, H&S

Unit-I: (Title)
LAPLACE TRANS FORMS

Important points / Definitions: (Minimum 15 to 20 points covering complete topics in that unit)

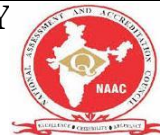
What is Laplace Transform? Formula, Properties, Conditions and Applications

Mathematics plays a decisive role to understand the behavior and working of electrical_and_electronic systems. Polynomials, Algebra, Probability, Integrations, and Differentiations etc...forms a significant part of the tools used to solve the systems. With the increasing complexity of systems, very sophisticated methods are required. Differential equations are prominently used for defining control systems. These equations are simple to solve. But complexity arises while solving higher order differential equations. To solve such complex higher order differential equations, the mathematical method that proved to be effective is Laplace Transform. As this transform is widely employed, it is useful to know what they really meant for and how do they work.

What is a Laplace Transform?

In mathematics, transforms are applied for transforming a variable from one form to another to make the equation easy to handle. Laplace transforms pretty much does the same thing. They transform higher order differential equation into a polynomial form which is far easy than solving differential equation directly

But there is various transform like Fourier transform, z transforms what makes Laplace transform special? The major advantage of Laplace transform is that, they are defined for both stable and unstable systems whereas Fourier transforms are defined only for stable systems.



Laplace Transform Formula

A Laplace transform of function f (t) in a time domain, where t is the real number greater than or equal

$$F(s) = \int_0^{\infty} f(t)e^{-st} dt$$

to zero, is given as F(s), where there s is the complex number in The

above equation is considered as unilateral Laplace transform equation. When the limits are extended

to the entire real axis then the Bilateral Laplace transform can be defined as

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-st} dt$$

In practical circuits like RC and RL circuits usually, initial conditions are used so, one-sided Laplace transforms are applied for analysis purpose.

As $s = \sigma + j\omega$, when $\sigma = 0$ Laplace transforms behaves as Fourier transform.

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\text{Re}\{s\} > 0$
3	$u(-t)$	$\frac{1}{s}$	$\text{Re}\{s\} < 0$
4	$\frac{t^{n-1}}{(n-1)!} u(t)$	$\frac{1}{s^n}$	$\text{Re}\{s\} < 0$
5	$-\frac{t^{n-1}}{(n-1)!} u(-t)$	$\frac{1}{s^n}$	$\text{Re}\{s\} < 0$
6	$e^{-\alpha t} u(t)$	$\frac{1}{s + \alpha}$	$\text{Re}\{s\} > -\alpha$
7	$-e^{-\alpha t} u(-t)$	$\frac{1}{s + \alpha}$	$\text{Re}\{s\} < -\alpha$
8	$\frac{t^{n-1}}{(n-1)!} e^{-\alpha t} u(t)$	$\frac{1}{(s + \alpha)^n}$	$\text{Re}\{s\} > -\alpha$
9	$-\frac{t^{n-1}}{(n-1)!} e^{-\alpha t} u(-t)$	$\frac{1}{(s + \alpha)^n}$	$\text{Re}\{s\} < -\alpha$
10	$\delta(t - T)$	e^{-sT}	All s
11	$[\cos \omega_0 t] u(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\text{Re}\{s\} > 0$
12	$[\sin \omega_0 t] u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\text{Re}\{s\} > 0$
13	$[e^{-\alpha t} \cos \omega_0 t] u(t)$	$\frac{s + \alpha}{(s + \alpha^2) + \omega_0^2}$	$\text{Re}\{s\} > -\alpha$



Conditions For Applicability of Laplace Transform

Laplace transforms are called integral transforms so there are necessary conditions for convergence of these transforms.

F(s) = integral from -infinity to infinity of f(t)e^-st dt < infinity: i.e. f must be locally integrable for the interval [0, infinity) and depending on whether sigma is positive or negative, e^(-sty) may be decaying or growing. For bilateral Laplace transforms rather than a single value the integral converges over a certain range of values known as Region of Convergence.

Properties of Laplace Transform:

Linearity

f1(t) L.T. -> F1(s) with ROC = R1

f2(t) L.T. -> F2(s) with ROC = R2

af1(t) + bf2(t) L.T. -> aF1(s) + bF2(s): ROC = R1 intersection R2 Linearity

Time Shifting

f(t) L.T. -> F(s) with ROC = R

f(t - t0) L.T. -> e^-st0 F(s): ROC = R

Time Shifting

Shift in S-domain

f(t) L.T. -> F(s) with ROC = R

e^st0 f(t) L.T. -> F(s - s0): ROC = R + Re{s0} Shift in S-domain

Time-reversal

f(t) L.T. -> F(s) with ROC = R

f(-t) L.T. -> F(-s) with ROC = -R

Time-reversal

Differentiation in S-domain

f(t) L.T. -> F(s) with ROC = R1

tf(t) L.T. -> -d/ds F(s): ROC = R

Differentiation in S-domain

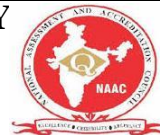
Convolution in Time

if f(t) L.T. -> F(s) with ROC = R1 and h(t) L.T. -> H(s) with ROC = R2

f(t) * h(t) L.T. -> F(s)H(s): ROC = R1 intersection R2

Convolution in Time Initial Value

Theorem



Initial value theorem is applied when in Laplace transform the degree of the numerator is less than the degree of

$$f(0) = \lim_{s \rightarrow \infty} sF(s)$$

the denominator

Final Value Theorem:

Short Questions (minimum 10 previous JNTUH Questions – Year to be mentioned)

UNIT-I

1. Define the Laplace transform (JNTU May 2006,2011,Dec 2012)
2. Define the Laplace transform of derivatives and integrals(JNTU JUNE 2010,2011)
3. Define the inverse Laplace transform(JNTU May 2008,,Dec 2011)
4. Find the Laplace transform of constant(JNTU May 2010,,Dec 2011)
5. State and prove second shifting theorem (JNTU JUNE 2007,2010,Dec 2011)
6. Using Laplace transform , solve + +5y=sint given that y(0)=0,(0)=1 (JNTUH2006)
7. Using Laplace transform , solve + +5y=constant given that y(0)=0,(0)=1(JNTUH2011)
- 8 State and prove first shifting theorem (JNTUH2008,JUNE2010)
- 9.State and prove linearity property of Laplace transform(JNTU May 2010,2011,Dec 2011)
- 10.prove that L(k)=k/s(JNTUH 2009,JUNE 2011)

Long Questions (minimum 10 previous JNTUH Questions – Year to be mentioned)

- 1.Find the Laplace transforms of e^{-t}(3 sin2t-5cosh2t)(JNTU2004,2007, june2010,dec 2011)
- 2.Find the Laplace transform of 3 cos4(t-2)-u(t-2) (JNTU Feb2012)
- 3.Evaluate L(tsin3tcos2t)(JNTU May 2010,2011,Dec 2011)
4. Find the Laplace transform of sin3tcost/t(JNTU May 2003,June 2012)
5. Find the L(cos4t sin2t/t)(JNTU June 2009,dec2010,2011)
- 6.Usng Inverse Laplace transform ,evaluate L⁻¹(1/s+1)(s²+1)(JNTU June 2008,dec2011,2012)
- 7.Find L⁻¹(s+3/s²-10s+29)(JNTU June 2005,dec2007,2013)
- 8.Find (cos⁻¹(s+2/3))(JNTU June 2012dec2013)
- 9.Slove the differential equation d²x/dx²+9x=sint using Laplace Transform given that x(0)=1,x(π/2)=1,x(0)=1,x¹(0)=0)(JNTU DEC2002,July 2011,2011June2012)
- 10.Using Laplace Transform ,solve (D²+1)x=tcost,given x=0,dy/dx=0,t=0(JNTU 200,7June 2010,2011June2012)

Fill in the Blanks / Choose the Best: (Minimum 10 to 15 with Answers)

- 1 .L{t sint }=2S/(S²+1)²_____
2. If f(t) is a periodic function if period T then L{(t)}=... ∫₀^α e^{-st}f(t)/(1-e^{-sT})
3. If H(t-a) is a unit step function {H(t-a)}=e^{-as}/s_____
4. The value of ∫₀[∞] te^{-3t} sint dt = 3/50_____
5. L{te^{at} sin at}= 2a(s-a)/((s-a)²+a²)²
6. L⁻¹{ 1 / ((s+1)(s+2)) }= e^{-t}-e^{-2t}
7. Solution of (D²+1)y=0 when y=1,Dy=0 when t=0 is = y=cost_____



8. The convolution $(f * g)(t)$ of f with g is defined by the equation $= \dots \int_0^t f(u) - g(t - u) du \dots$

9. $L^{-1} \{ \frac{1}{s^n} \}$ is possible when n is = positive integer

10. $L^{-1} \{ \frac{1}{s} \} = \underline{1}$

Unit-II: (Title)
NUMERICAL METHODS-1

Important points / Definitions: (Minimum 15 to 20 points covering complete topics in that unit)

Bisection Method:

This is one of the simplest and reliable iterative methods for the solution of nonlinear equation. This method is also known as binary chopping or half-interval method. Given a function $f(x)$ which is real

and continuous in an interval $[a, b]$ and $f(a)$ and $f(b)$ are of opposite sign i.e. $f(a)f(b) < 0$, then there is at least one real root $\xi \in [a, b]$ of $f(x) = 0$.

Algorithm:

Given a function $f(x)$ continuous on a interval $[a, b]$ satisfying the Bisection method starting criteria, carry out the following steps to find a root ξ of $f(x)$

(1) Set $a_0 = a, b_0 = b$

(2) For $n=1,2,\dots$ until satisfied do

(a) If $f\left(\frac{a_{n-1} + b_{n-1}}{2}\right) = 0$ then $\xi = \frac{a_{n-1} + b_{n-1}}{2}$

(b) If $f(a_{n-1})f\left(\frac{a_{n-1} + b_{n-1}}{2}\right) < 0$ then

set $a_n = a_{n-1}; b_n = \frac{a_{n-1} + b_{n-1}}{2}$

otherwise

set $a_n = \frac{a_{n-1} + b_{n-1}}{2}; b_n = b_{n-1}$

Note:

1) The subscripts in $a_0, b_0, a_{n-1}, b_{n-1}, a_n, b_n$ etc denote the iteration number. $[a_0, b_0]$ is the interval for the zeroth or starting iteration. $[a_n, b_n]$ is the interval for the n^{th} iteration.



(2) An iterative process must be terminated at some stage. 'Until satisfied' refers to the solution convergence criteria used for stopping the execution process. We must have an objective criteria for deciding when to stop the process. We may use one of the following criteria depending on the behavior of the function (monotonous/steep variation/increasing /decreasing)

- (i) $|x_{n+1} - x_n| < \epsilon_a$ (Tolerable absolute error in ξ)
- (ii) $\left| \frac{x_{n+1} - x_n}{x_{n+1}} \right| < \epsilon_r$ (Tolerable relative error in ξ)
- (iii) $|f(x_{n+1})| < \epsilon$ (Value of function as $x_n \rightarrow \xi$)
- (iv) $|f(x_{n+1}) - f(x_n)| < \epsilon$ (difference in two consecutive iteration function values)

Usually $\epsilon_a, \epsilon_r, \epsilon$ are referred to as tolerance values and it is fixed by us depending on the level of accuracy we desire to have on the solution. For example $\epsilon_a \approx 10^{-70}, \epsilon_r \approx 10^{-12}, \epsilon \approx 10^{-10}$ etc.

Example:

$$2x^3 - 2.5x - 5 = 0$$

Solve for the root in the interval [1,2] by Bisection method.

Solution: Given $f(x) = 2x^3 - 2.5x - 5$ on [1, 2]

$$f(1) = -5.5 < 0$$

$$f(2) = 6 > 0$$

$$f(1)f(2) = -33 < 0$$

∴ There is a root for the given function in [1,2].

$$a_0 = 1, \quad b_0 = 2, \quad \epsilon \approx 10^{-6}$$

Set

$$f\left(\frac{a_0 + b_0}{2}\right) = f\left(\frac{1+2}{2}\right) = f(1.5) = -2.0$$

$$\therefore f(a_0)f\left(\frac{a_0 + b_0}{2}\right) = 11 > 0 \quad ; \quad \left|f\left(\frac{a_0 + b_0}{2}\right)\right| > \epsilon$$

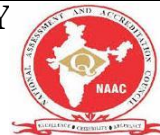
$$a_1 = \frac{a_0 + b_0}{2} = 1.5 \quad ; \quad b_1 = b_0 = 2$$

Set

$$f\left(\frac{a_1 + b_1}{2}\right) = f\left(\frac{1.5+2}{2}\right) = f(1.75) = 1.34375 > 0$$

$$\therefore f(a_1)f\left(\frac{a_1 + b_1}{2}\right) = -2.6875 < 0 \quad \text{and} \quad |f(1.75)| > \epsilon$$

$$a_2 = a_1 = 1.5 \quad ; \quad b_2 = \frac{a_1 + b_1}{2} = 1.75$$



False Position or Regular False method:

Bisection method converges slowly. Here while defining the new interval $[a_n, b_n]$ the only utilization of the function $f(x)$ is in checking whether $f(a_n)f(b_n) < 0$ but not in actually calculating the end point of the interval. False Position or Regular Falsi method uses $f(x)$ not only in deciding the new interval $[a_n, b_n]$ as in bisection method but also in calculating one of the end points of the new interval.

Here one of end points of $[a_n, b_n]$ say w is calculated as a weighted average defined on previous interval $[a_{n-1}, b_{n-1}]$ as

$$w = \frac{|f(b_{n-1})|a_{n-1} + |f(a_{n-1})|b_{n-1}}{|f(b_{n-1})| + |f(a_{n-1})|}$$

$$= \frac{f(b_{n-1}) a_{n-1} - f(a_{n-1}) b_{n-1}}{f(b_{n-1}) - f(a_{n-1})}$$

$\because f(a_{n-1}), f(b_{n-1})$ have opposite signs.

The algorithm for computing the root of function $f(x)$ by this method is given below.

Algorithm:

Given a function $f(x)$ continuous on an interval $[a, b]$ satisfying the criteria $f(a)f(b) < 0$, carry out the following steps to find the root ξ of $f(x)$ in $[a, b]$:

- (1) Set $a_0 = a, b_0 = b$
- (2) For $n = 0, 1, 2, \dots$ until convergence criteria is satisfied, do:

(a) Compute
$$= \frac{f(2) \cdot 1 - f(1) \cdot 2}{f(2) - f(1)} = 1.47826087$$

(b) If $(f(a_n)f(w) \leq 0)$, then set $a_{n+1} = a_n ; b_{n+1} = w$

otherwise set $a_{n+1} = w ; b_{n+1} = b_n$

Note:

Use any one of the convergence criteria discussed earlier under bisection method. For the sake of carrying out a comparative study we will stick both to the same convergence criteria as before

$|f(\xi_n)| = |f(w)| < \epsilon = 10^{-6}$
 i.e. (say) and to the example problems.

Example:

$2x^3 - 2.5x - 5 = 0$
 Solve for the root in the interval $[1, 2]$ by Regula-Falsi method:

Solution: Since $f(1)f(2) = -33 < 0$, we go ahead in finding the root of given function $f(x)$ in $[1, 2]$.

Set $a_0 = 1, b_0 = 2$

$$= \frac{f(2) \cdot 1 - f(1) \cdot 2}{f(2) - f(1)} = 1.47826087 \qquad = \frac{f(2) \cdot 1 - f(1) \cdot 2}{f(2) - f(1)} = 1.47826087$$

$f(w) = f(1.47826087) = -2.23489761$



∴ f(a0)f(w) = (-5.5) × (-2.23489761) > 0

set a1 = w = 1.47826087 ; b1 = b0 = 2

∴ |f(w)| > ε = 10^-6, proceed with iteration.

Newton-Raphson Method:

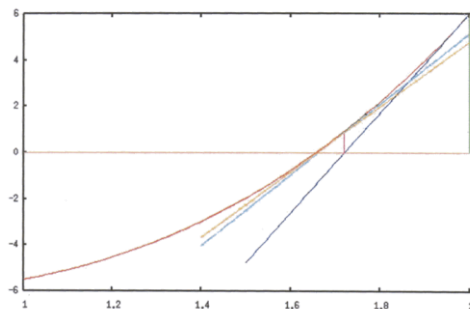
Unlike the earlier methods, this method requires only one appropriate starting point x0 as an initial assumption of the root of the function f(x) = 0. At (x0, f(x0)) a tangent to f(x) = 0 is drawn. Equation of this tangent is given by

y = f'(x0)(x - x0) + f(x0)

The point of intersection, say x1, of this tangent with x-axis (y = 0) is taken to be the next approximation to the root of f(x) = 0. So on substituting y = 0 in the tangent equation we get

x1 = x0 - f(x0)/f'(x0)

If |f(x1)| < ε = 10^-6 (say) we have got an acceptable approximate root of f(x) = 0, otherwise we replace x0 by x1, and draw a tangent to (x1, f(x1)) and consider its intersection, say x2, with x-axis as an improved approximation to the root of f(x)=0. If |f(x2)| > ε, we iterate the above process till the convergence criteria is satisfied. This geometrical description of the method may be clearly visualized in the figure below:



The various steps involved in calculating the root of f(x)=0 by Newton Raphson Method are described compactly in the algorithm below.

Algorithm:

Given a continuously differentiable function f(x) and an initial approximation x0 to the root of f(x) = 0, the steps involved in calculating an approximation to the root of f(x) = 0 s.t. |f(ξ)| < ε are:

- (1) Calculate f(x0), f'(x0) and set ε = ε-tilde
(2) For n = 0,1,2... until convergence criteria is satisfied, do: Calculate



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Remark (1): This method converges faster than the earlier methods. In fact the method converges at a quadratic rate. We will prove this later.

Remark (2): This method can be derived directly by the Taylor expansion $f(x)$ in the neighborhood of the root ξ of $f(x) = 0$. The starting approximation x_0 to ξ is to be properly chosen so that the first order Taylor series approximation of $f(x_0 + h)$ in the neighborhood of x_0 leads to x_1 , an improved approximation to ξ . i.e

$$f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h^2}{2}f''(x_0) + \dots = 0$$

$\therefore h \ll 1$, neglecting h^2 and its higher powers, we get
 $f(x_0) + hf'(x_0) = 0$

i.e.

$$h = -\frac{f(x_0)}{f'(x_0)}, \quad \therefore h = x_1 - x_0$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Now the successive approximations $x_2, x_3, \dots, x_n, \dots$ etc may be calculated by the iterative formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Remark(3) : One may also derive the above iteration formulation starting with the iteration formula for the secant method. In a way this may help one to visualize Newton-Raphson method as an improvement over the secant method. So, let us consider the iteration formula for the secant method i.e.

$$x_{n+1} = \frac{f(x_n)x_{n-1} - f(x_{n-1})x_n}{f(x_n) - f(x_{n-1})}$$

Add and subtract $f(x_n)x_n$ to the numerator on the R.H.S. to get

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{\frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}}$$

or ,



Clearly, $\frac{f(x_n) - f(x_{n-1})}{(x_n - x_{n-1})}$ is the slope of the secant to the curve $f(x) = 0$ through the points $(x_{n-1}, f(x_{n-1}))$ and $(x_n, f(x_n))$. This also represents slope of the tangent to $f(x)=0$ parallel to the secant intersecting x-axis between x_{n-1} and x_n . If $f(x)$ is differentiable one may as well approximate this slope by $f'(x_n)$ and thus arrive at the iteration formula.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

NEWTON- FORWARD METHIOD

Newton Divided Difference Table:

It may also be noted for calculating the higher order divided differences we have used lower order divided differences. In fact starting from the given zeroth order differences $f[x_i]$; $i = 0,1,\dots,n$, one can systematically arrive at any of higher order divided differences. For clarity the entire calculation may be depicted in the form of a table called

Newton Divided Difference Table.

i	x_i	$f[x_i]$	First order differences	Second order differences	Third order differences	Fourth order differences	Fifth order differences
0	x_0	$f[x_0]$	$f[x_0, x_1]$	$f[x_0, x_1, x_2]$	$f[x_0, x_1, x_2, x_3]$	$f[x_0, x_1, x_2, x_3, x_4]$	$f[x_0, x_1, x_2, x_3, x_4, x_5]$
1	x_1	$f[x_1]$					
2	x_2	$f[x_2]$	$f[x_1, x_2]$				
3	x_3	$f[x_3]$	$f[x_2, x_3]$	$f[x_1, x_2, x_3]$			
4	x_4	$f[x_4]$	$f[x_3, x_4]$	$f[x_2, x_3, x_4]$	$f[x_1, x_2, x_3, x_4]$		
5	x_5	$f[x_5]$	$f[x_4, x_5]$	$f[x_3, x_4, x_5]$	$f[x_2, x_3, x_4, x_5]$		

Again suppose that we are given the data set (x_i, f_i) , $i = 0 \dots 5$ and that we are interested in finding the 5th order Newton Divided Difference interpolynomial. Let us first construct the Newton Divided Difference Table. Wherein one can clearly see how the lower order differences are used in calculating the higher order Divided Differences:

Example:

Construct the Newton Divided Difference Table for generating Newton interpolation polynomial with the following data set:

i	0	1	2	3	4
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x_i	0	1	2	3	4
$y_i = f(x_i)$	0	1	8	27	64

Solution:

Here $n=5$. One can fit a fourth order Newton Divided Difference interpolation polynomial to the given data. Let us generate Newton Divided Difference Table; as requested.

i	x_i	$f[x_i]$	1 st order differences	2 nd order differences	3 rd order differences	4 th order differences
0	0	0				
1	1	1	$\frac{1-0}{1-0} = 1$			
2	2	8	$\frac{8-1}{2-1} = 7$	$\frac{7-1}{2-0} = 3$		
3	3	27	$\frac{27-8}{3-2} = 19$	$\frac{19-7}{3-1} = 6$	$\frac{6-3}{3-0} = 1$	
4	4	64	$\frac{64-27}{4-3} = 37$	$\frac{37-19}{4-2} = 9$	$\frac{9-6}{4-1} = 1$	$\frac{1-1}{4-0} = 0$

Note: One may note that the given data corresponds to the cubic polynomial x^3 . To fit such a data 3rd order polynomial is adequate. From the Newton Divided Difference table we notice that the fourth order difference is zero. Further the divided differences in the table

Newton-Gregory Backward Difference

Newton Interpolation polynomial with equidistant points:

Gregory-Newton Forward Difference Approach:

$$(x_i, y_i), i = 0, 1, \dots, n$$

Very often it so happens in practice that the given data set $(x_i, y_i), i = 0, 1, \dots, n$ correspond to a sequence $\{x_i\}$ of equally spaced points. Here we can assume that

$$x_i = x_0 + ih, \quad i = 0, 1, 2 \dots n$$

where x_0 is the starting point (sometimes, for convenience, the middle data point is taken as x_0 and in such a case the integer i is allowed to take both negative and positive values.) and h is the step size. Further it is enough to calculate simple differences rather than the divided differences as in the



non-uniformly placed data set case. These simple differences can be forward differences (Δf_i) or backward differences (∇f_i) . We will first look at forward differences and the interpolation polynomial based on forward differences.

The first order forward difference Δf_i is defined as
$$\Delta f_i = f_{i+1} - f_i$$

The second order forward difference $\Delta^2 f_i$ is defined as
$$\Delta^2 f_i = \Delta f_{i+1} - \Delta f_i$$

The k^{th} order forward difference $\Delta^k f_i$ is defined as
$$\Delta^k f_i = \Delta^{k-1} f_{i+1} - \Delta^{k-1} f_i$$

Since we already know Newton interpolation polynomial in terms of divided differences, to derive or generate Newton interpolation polynomial in terms of forward differences it is enough to express forward differences in terms of divided differences.

Recall the definition of first divided difference $f[x_0, x_1]$,

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f_1 - f_0}{h} = \frac{\Delta f_0}{h}$$

$$\therefore \Delta f_0 = hf[x_0, x_1]$$

Similarly we can get

$$\Delta f_1 = hf[x_1, x_2]$$

by the definition of second order forward difference $\Delta^2 f_0$, we get

$$\Delta^2 f_0 = \Delta f_1 - \Delta f_0$$

$$= hf[x_1, x_2] - hf[x_0, x_1] \quad (\text{using (8.1) \& (8.2)})$$

$$= h\{f[x_1, x_2] - f[x_0, x_1]\}$$

$$= h \cdot 2h\{(f[x_1, x_2] - f[x_0, x_1])/2h\}$$

$$= 2h^2\{(f[x_1, x_2] - f[x_0, x_1])/(x_2 - x_0)\}$$



In a similar way, in general, we can show that

$$\Delta^k f_i = k!h^k f[x_i, x_{i+1}, x_{i+2} \dots x_{i+k}]$$

$$\therefore f[x_i, x_{i+1}, \dots x_{i+k}] = \frac{\Delta^k f_i}{k!h^k}$$

$$f[x_0, x_1 \dots x_k] = \frac{\Delta^k f_0}{k!h^k}$$

Now using (6.1) & (8.6) the Newton forward difference interpolation polynomial may be written as follows:

$$P_n(x) = \sum_{k=0}^n \frac{\Delta^k f_0}{k!h^k} \prod_{i=0}^{k-1} (x - x_i)$$

To rewrite (9) in a simpler way let us set

$$x = x_0 + sh, \quad P_n(s) = P_n(x)$$

$$\therefore x_k = x_0 + kh$$

$$x - x_k = (s - k)h$$

$$\begin{aligned} \therefore P_n(s) &= \sum_{k=0}^n \frac{\Delta^k f_0}{k!h^k} \prod_{i=0}^{k-1} (s - i)h \\ &= \sum_{k=0}^n \frac{\Delta^k f_0}{k!h^k} [s(s-1)\dots(s-k+1)]h^k \end{aligned}$$

i.e
$$P_n(s) = \sum_{k=0}^n \binom{s}{k} \Delta^k f_0$$

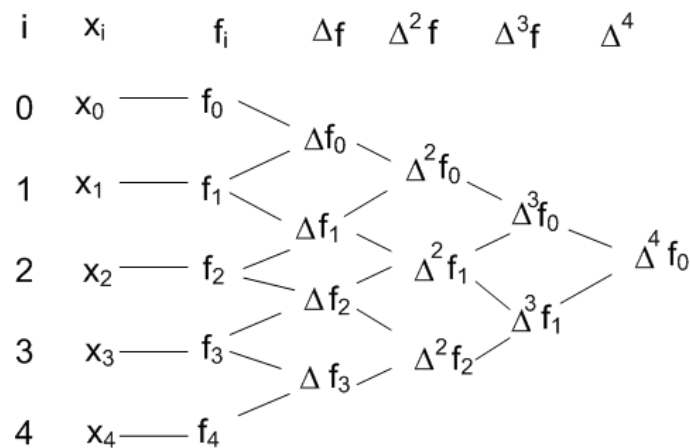
where

$$\binom{s}{k} = \frac{s(s-1)\dots(s-k+1)}{k!}$$

This is known as Newton-Gregory forward difference interpolation polynomial. For convenience while constructing (10) one can first generate a forward difference table and use the $\Delta^k f_i$ from the table.



Suppose we have data set $(x_i, f_i) \quad i = 0, 1, 2, 3, 4$ then forward difference table looks as follows:



Short Questions (minimum 10 previous JNTUH Questions – Year to be mentioned)

- 1).Find the second difference of the polynomial $x^4-12x^3+42x^2-30x+9$ with interval of difference $h=2$ (JNTU May 2007,Dec 2011)
- 2.If he interval difference is unity prove that $\Delta(x(x+1)(x+2)(x+3))=4(x+1)(x+2)(x+3)$ (JNTU May 2010,Dec 2013)
- 3) Evaluate $f(10)$ given $f(x)=168,192,336$ at $x=1,7,15$ respectively use Lagrange interpolation
- 4).Find the unique polynomial $p(x)$ (JNTU May 2008,2011,Dec 2014)
- 5).Find a positive root of the equation $x^3-4x-9=0$ using bisection method in four stages(JNTU Sep 2008)
- 6)Find a real root of $x^3-x-2=0$ using Iterative method in to 3 decimal points(JNTUH 2008,2009)
7. Slove the equation $x^3+x^2-1=0$ for a positive root by iteration method(JNTUH 2008)
- 8.Find a real root of the equation $3x-\cos x -1=0$ using Newton Raphson method(JNTUH June 2012)
9. Find a real root of the equation $x^3-x-2=0$ using Newton Raphson method(JNTUH June 2010)
10. Slove the equation $9x^3+5x^2-1=0$ for a positive root by iteration method(JNTUH 2009)

Long Questions (minimum 10 previous JNTUH Questions – Year to be mentioned)

- 1).Find a root of $e^x \sin(x)=1$ using Newton Raphson Method(JNTU May 2010,Dec 2011)
- 2)Find a positive root of $x^3-x-1=0$ correct to two decimal points by bisection method(June 2009,June 2010,June 2011)
- 3.Using bisection method ,find the negative root of $x^3-4x+9=0$ (JNTUH June 2009)
- 4.Find a real root of the equation $\cos x-x^2-x=0$ using Newton Rapshon method(June 2012)
- 5.Find a real root of the equation $3x-\cos x-1=0$ using Newton Rapshon method(JNTUH 2012)
- 6).Find $f(2.5)$ using Now tons formula from the following table (JNTU May 2006,2010)

X	0	1	2	3	4	5	6
Y	0	1	16	81	256	625	1296

- 7).Find $y(54)$ given that $y(50)=205, y(60)=225, y(70)=248$ and $y(80)=274$ using Now tons Forward difference (JNTU May 2006, June 2014)
- 8)..Find the polynomial which fits the data in the following table using Gauss forward formula (JNTU May 2010,Dec 2016)



X	3	5	7	9	4
Y	6	24	58	108	174

9).Using Gauss back word difference formula find y(8) form following table (JNTU May 2012,Dec2016)

X	0	5	10	5	20	25
Y	7	1	14	18	24	12

10). Using Lagrange's formula find y(8)(JNTU May 2014,June 2017)

X	3	5	7	9	11
Y	6	24	58	108	74

Fill in the Blanks / Choose the Best: (Minimum 10 to 15 with Answers)

- 1.The Bisection Method to find the roots of an equation $f(x)=0$ are ---equal-----
- 2.The order of convergence of Method of Successive Approximations---are same-----
- 3.Out of Regular False Method and Newton Rap son Method, the rate of convergence is faster for ---Regula False-----
- 4.The condition for Iteration Method is --Tow approximations are same-----
- 5..Formula to find the roots of an equation in Newton Rap son's method is-.... $x_i \cdot f(x_i) / f'(x_i)$ -----
- 6.The order of convergence in New ton Rap sons method is.....2.....
- 7.Incase of bisection method ,the convergence is.... Very slow.....
- 8..Lagranges....formula is used for unequal .intervals of x values
- 9.Gauss-Backword interpolation formula i used to interpolate the values of y for.-
 $1 < p < 0$
- 10.the relation between E end noble is ... $1 - E^{-1}$



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